EXCITATION OF TOLLMIEN—SCHLICHTING WAVES BY ACOUSTIC AND VORTEX DISTURBANCE SCATTERING IN BOUNDARY LAYER ON A WAVY SURFACE

N. A. Zavol'skii, V. P. Reutov, and G. V. Rybushkina

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It is well known that the excitation of Tollmien—Schlichting waves by acoustic and vortex fields can appreciably influence transition to turbulence in a boundary layer. A large number of studies is devoted to the investigation of the excitation process of Tollmien—Schlichting waves (TS-waves) in the boundary layer on a smooth flat plate (see [1, 2]). Laboratory and numerical experiments showed that boundary layer waves appear in the neighborhood of the plate leading edge [1, 3, 4]. The onset of TS-waves can be considered as the result of scattering of the external field on concentrated roughness, viz, the plate leading edge. Distributed excitation was not observed on the plate itself [3]. The reason for this is that distributed excitation on a smooth plate can be caused only by linear interaction of external disturbances with the boundary layer waves which is ineffective in view of the large difference in phase velocities. The excitation of TS-waves in the presence of scattering of acoustic and vortex disturbances in the boundary layer on a surface with distributed waviness, whose characteristic wavelength in the streamwise direction is less than or is of the order of TS-waves is studied in this paper. The principle of the scattering process lies in coincident harmonic summation of the spatial spectrum of the roughness with the harmonics of the external field. In this case coincident "forces" could occur both in the boundary layer and on the plate surface which could be in resonance with the induced wave in spite of the absence of resonance of this wave with the external field. In the case of distributed roughness, spatial accumulation of the scattering process takes place (distributed generation). The interest in the given mechanism is for a number of reasons. Firstly, after the onset of the wave at the plate leading edge it manages to get strongly damped in the passive segment of the boundary layer [4]. In scattering over distributed roughness, disturbances are carried directly into the active area of the boundary layer which creates an advantage in terms of excitation effectiveness. Secondly, there is no generation near the leading edge with the superposition of streamwise acoustic field or vortex disturbances, localized outside the boundary layer [1]. The distributed mechanism does not have such "sensitivity" to the type of disturbance. The effectiveness of scattering is estimated for just these two types of external disturbances. Computations are carried out for one-dimensional sinusoidal and random waviness.

1. Consider a Blasius boundary layer on a surface with a small waviness described by a single-valued function $y = \eta(x, z)$. We use the quasiparallel model for the boundary-layer flow, expressing velocity field in the form $\mathbf{v} = U(y/\delta_*)\mathbf{i} + \mathbf{v}_\sim$, where U is the velocity profile in the boundary layer on the flat plate surface $y = 0$; $\delta_*(x)$ is the displacement thickness; $\mathbf{v}_\sim$ is the disturbance in the initial flow. Using series expansion in terms of small η, the boundary condition $v = 0|_{y=\eta}$ can be reduced to the following relations on the surface $y = 0$:

$$v + \frac{\partial v}{\partial y}\eta + \ldots = 0, \quad u + \frac{\partial U}{\partial y}\eta + \frac{\partial u}{\partial y}\eta + \ldots = 0,$$
$$w + \frac{\partial w}{\partial y}\eta + \ldots = 0 \quad \text{for} \quad y = 0, \tag{1.1}$$

where u, v, and w are the components of $\mathbf{v}_\sim$ along x, y, and z, respectively. Boundary conditions in form (1.1) make it possible to consider the roughness as an external "force" within the framework of the boundary-layer model on a flat surface.

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The disturbance velocity $\mathbf{v}_i$ and pressure p_i caused by waviness is expressed in the form of a series in powers of η. Total velocity and pressure disturbances are sought in the form of a series in terms of scattering frequency [5]:

$$\mathbf{v}_\sim = \mathbf{v}_i + \mathbf{v}^{(0)} + \mathbf{v}^{(1)} + \ldots, \quad p_\sim = p_i + p^{(0)} + p^{(1)} + \ldots,$$

where $\mathbf{v}^{(0)}$, $p^{(0)}$ is the field in the boundary layer on a smooth surface; $\mathbf{v}^{(n)}$, $p^{(n)}$ ($n = 1, 2, \ldots$) are the components of the scattering field $\sim \eta^n$. Further analysis is limited to single frequency scattering when the fundamental contributions to the scattered field are made by $\mathbf{v}^{(1)}$ and $p^{(1)}$. The boundaries for the applicability of single frequency scattering approximation will be determined in Section 3. We specify acoustic and vortex disturbances in the form of sinusoidal waves $\sim \mathrm{Re}[\exp(ik_\omega x - i\omega t)]$ (ω and k are the frequency and wave number, respectively). Interacting with the harmonics of distributed roughness $\sim \exp(ik_x x + ik_z z)$ (α is the wave number of three dimensional TS-waves (ω, k_z)). The most effective scattering is made by the harmonics of roughness satisfying the resonance condition

$$k_x = \mathrm{Re}\,\alpha - k_\omega. \tag{1.2}$$

If the roughness spectrum is continuous and two-dimensional, the condition (1.2) can be satisfied for a wide range of values of k_z. It follows from (1.2) that acoustic scattering in the subsonic boundary layer takes place on the components of the roughness spectrum $k_x \approx \mathrm{Re}\,\alpha$.

Let $x_c = x_c(\omega, k_z)$ be the critical point for the induced wave. In the region $x < x_c$ the generated waves are damped and hence are not capable of competing with those generated at $x \approx x_c$. The same disturbances that are created in the unstable region give way to neutral disturbances along the growth length. It follows that the boundary-layer region adjacent to the critical point is most sensitive to the distributed action.

2. Consider acoustic scattering on one-dimensional waviness $\eta = \eta(x)$ which excites two-dimensional TS waves. The basic equations are for a barotropic viscous fluid. The velocity field resulting from the flow past the wavy surface is described by the stream function $\psi(x, y) = \psi^{(1)} + \psi^{(2)} + \ldots$ ($\psi^{(n)}$ ($n = 1, 2, \ldots$) are the components $\sim\eta^n$). Neglecting the dependence of δ_* on x switching over to spectral representation using the equation

$$\hat{\psi} = \int_{-\infty}^{\infty} \psi(x, y)\, e^{-ikx} dx,$$

we get for $\bar{\psi} = -\hat{\psi}^{(1)}/\hat{\eta}(\partial U/\partial y)_0$ ($\hat{\eta}(k)$ is the spectrum of $\eta(x)$) an equation of the type

$$\bar{u}(y_N)(\bar{\psi}'' - k_N^2\bar{\psi}) - \bar{u}''\bar{\psi} + \frac{i}{k_N \mathrm{R}}(\bar{\psi}^{\mathrm{IV}} - 2k_N^2\bar{\psi}'' + k_N^4\bar{\psi}) = 0, \tag{2.1}$$

where $\mathrm{R} = u_\infty\delta_*/\nu$ is the local Reynolds number (u_∞ is the free stream velocity, ν is the kinematic viscosity); $\bar{u} = U/u_\infty$; $y_N = y/\delta_*$: $k_N = k\delta_*$ (here and in what follows, the index N denotes variables normalized with respect to displacement thickness and prime denotes differentiation with respect to y_N). The variable $\bar{\psi}$ does not depend on η, since boundary conditions follow from (1.1) $\bar{\psi} = 0$, $\bar{\psi}' = 1|_{y_N = 0}$; as $y_N \to \infty$, as usual, the condition $\bar{\psi} \to 0$ is introduced. Equation (2.1) is a particular case of the Orr–Sommerfeld equation which is well known in hydrodynamic stability theory [6]. In the present case this equation generates the nonhomogeneous boundary-value problem for the disturbance profile whose phase velocity is zero.

Acoustic field is specified in the form of the fundamental mode of the waveguide, one of whose boundaries is the plate surface and the other is outside the boundary layer. Neglecting the fluid motion and acoustic damping over the region of wave generation, the acoustic field is written in the form

$$(u, v, p) = \frac{1}{2} s(e_1, e_2, e_3)\, e^{ik_\omega x - i\omega t} + \mathrm{c.c.} \tag{2.2}$$

where $e_1 = 1 - \exp(\beta y)$; $e_2 = -(ik_\omega/\beta)[1 - \exp(\beta y)]$; $e_3 = \pm c/\rho$ (ρ is the fluid density); $k_\omega = \pm\omega/c$; c is the speed of sound, signs $\pm$ refer to waves propagating downstream and up-stream, respectively; $\beta = (i-1)\sqrt{\omega/2\nu}$. The thickness of the viscous wall layer in the acoustic field $l_W = \sqrt{2\nu/\omega}$ with excited TS-waves usually satisfies the condition $l_W \ll \delta_*$.

Equations for the scattered field take the form

$$\frac{\partial u^{(1)}}{\partial t} + U\frac{\partial u^{(1)}}{\partial x} + \frac{\partial U}{\partial y}v^{(1)} + \frac{1}{\rho}\frac{\partial p^{(1)}}{\partial x} + \nu\left(\frac{\partial^2 u^{(1)}}{\partial x^2} + \frac{\partial^2 u^{(1)}}{\partial y^2}\right) = f_1,$$
$$\frac{\partial v^{(1)}}{\partial t} + U\frac{\partial v^{(1)}}{\partial x} + \frac{1}{\rho}\frac{\partial p^{(1)}}{\partial x} + \nu\left(\frac{\partial^2 v^{(1)}}{\partial x^2} + \frac{\partial^2 v^{(1)}}{\partial y^2}\right) = f_2, \quad \frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y} = f_3, \tag{2.3}$$

where

$$f_1 = -\frac{\tilde{p}}{c^2\rho}\frac{\partial\tilde{u}}{\partial t} - \tilde{u}\frac{\partial\tilde{u}}{\partial x} - \tilde{v}\frac{\partial\tilde{u}}{\partial y}; \quad f_2 = -\frac{\tilde{p}}{c^2\rho}\frac{\partial\tilde{v}}{\partial t} - \tilde{v}\frac{\partial\tilde{v}}{\partial y} - \tilde{u}\frac{\partial\tilde{v}}{\partial x};$$
$$f_3 = \frac{(n-1)}{c^2\rho}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\tilde{p}^2 - \frac{1}{c^2\rho}\left(\frac{\partial\tilde{p}\tilde{u}}{\partial x} + \frac{\partial\tilde{p}\tilde{v}}{\partial y}\right); \tag{2.4}$$

($n \sim 1$ is the nonlinear parameter of the medium [7]). The variables $\tilde{u}$, $\tilde{v}$, and $\tilde{p}$ in (2.4) are the superpositions of acoustic waves (2.2) and the flow field past wavy surface; in accordance with the procedure for the elimination of f_i from (2.3) it is necessary to exclude terms depending only on v_i, p_i. Boundary conditions at $y = 0$ have the form $v^{(1)} = -(\partial v^{(0)}/\partial y)\eta$, $u^{(1)} = -(\partial u^{(0)}/\partial y)\eta$.

Consider distributed excitation of TS-waves with frequency of sound ω (the scale of the excitation region $L_* \gg 1/\mathrm{Re}\,\alpha$). Following quasiparallel flow model we shall completely neglect the derivatives of δ_* with respect to x. Switching over from system (2.3) to one equation for $v^{(1)}$, we represent its solution in the form

$$v^{(1)} = \frac{1}{2}a(x)\varphi(y_N)e^{i\theta - i\omega t} + \text{к. с.} + \delta v, \tag{2.5}$$

where $\theta = \int \mathrm{Re}\,\alpha dx$; $a(x)$ is the complex wave amplitude (change in $a(x)$ is small as θ changes by 2π); δv is the additional term of the order $(L_*\mathrm{Re}\,\alpha)^{-1} \ll 1$; α and φ are the eigenvalue and the eigenfunction of the boundary-value problem

$$\left(\bar{u} - \frac{\omega_N}{\alpha_N}\right)(\varphi'' - \alpha_N^2\varphi) - \bar{u}''\varphi + \frac{i}{\alpha_N \mathrm{R}}(\varphi^{\mathrm{IV}} - 2\alpha_N^2\varphi'' + \alpha_N^4\varphi) = 0,$$
$$\varphi = \varphi' = 0|_{y_N=0}, \quad \varphi \to 0|_{y_N\to\infty}, \tag{2.6}$$

where $\alpha_N = \omega\delta_*$, $\omega_N = \omega\delta_*/u_\infty$. The derivative da/dx is sought in the form

$$da/dx = \gamma a + F^{(1)}, \tag{2.7}$$

where $\gamma = -\mathrm{Im}\,\alpha$ is the increment in spatial growth of the wave; $F^{(1)}$ is the unknown function which is determined from the condition for the boundedness of δv. Using the standard procedure [8], we get

$$F^{(1)} = \frac{1}{2\pi}s\sigma\int_{-\infty}^{\infty} dk\hat{\eta}(k)\,e^{i(k+k_\omega)x - i\theta}, \tag{2.8}$$

where

$$\sigma = \sigma_N/\delta_*^2;\ \sigma_N = \sigma_S + \sigma_V;\ \sigma_V = \frac{1}{q}\int_0^\infty Q\chi dy_N;\ \sigma_S = \frac{i\alpha_N\beta_N}{q\mathrm{R}}\chi''(0);$$
$$Q = \alpha_N^2\bar{u}_0'\left[(1 - e^{\beta_N y_N})(\bar{\psi}'' - \bar{k}_N^2\bar{\psi}) - i\omega_N \mathrm{R}\bar{\psi}e^{\beta_N y_N}\right],$$
$$q = \int_0^\infty \left[\left(\bar{u} - \frac{4i\alpha_N}{\mathrm{R}}\right)\varphi'' + \left(2\omega_N\alpha_N - 3\alpha_N^2\bar{u} - \bar{u}'' + \frac{4i\alpha_N^3}{\mathrm{R}}\right)\varphi\right]\chi dy_N. \tag{2.9}$$

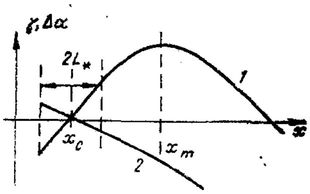

Fig. 1

Here $\beta_N = (i-1)\sqrt{\omega_N R/2}$; $\chi(y_N)$ is the eigenfunction of the conjugate boundary-value problem (2.6) [9]; $\bar{\psi}$ is the solution to (2.1) for the resonant component of the roughness spectrum $\bar{k} = \mathrm{Re}\,\alpha - k_\omega$ ($\bar{k}_N = \bar{k}\delta_*$)†. In computing the scatter coefficient σ the Mach number $M = u_\infty/c$ was assumed to be small and quantities of the order O(M) were neglected. The terms σ_v and σ_s describe volumetric and surface scattering, respectively. It is worth noting that in the present approximation σ depends only on the streamwise velocity component in the acoustic field and does not depend on the direction of sound propagation.

The problem of the excitation of TS-waves by vortex disturbances also leads to Eqs. (2.3). Consider the scattering of weak vortex disturbances drifting into the region of uniform flow (y > 3). Their stream function is represented in the form

$$\bar{\psi} = \begin{cases} \frac{1}{2} s\Phi(y)\, e^{ik_\omega x - i\omega t} + \text{к. с.}, & y > 3, \\ 0, & 0 \leqslant y \leqslant 3, \end{cases} \tag{2.10}$$

where $k_\omega = \omega/u_\infty$; Φ and s are the vortex wave profile and amplitude, respectively. Neglecting viscous vortex diffusion over lengths $2L_*$, the profile Φ may be specified arbitrarily. There is no surface scattering of disturbances (2.10) ($\sigma_s = 0$), and the spatial scattering is determined by the coefficient σ_v. In computing the latter from (2.9) it is necessary to put

$$Q = \bar{k}_N \bar{u}_0' \left\{ \omega_N \left[\bar{\psi}' (\Phi'' - \omega_N^2 \Phi) - \Phi \frac{\partial}{\partial y_N} (\bar{\psi}'' - \bar{k}_N^2 \bar{\psi}) \right] - \bar{k}_N \left[\bar{\psi} \frac{\partial}{\partial y_N} (\Phi'' - \omega_N^2 \Phi) - \Phi' (\bar{\psi}'' - \bar{k}_N^2 \bar{\psi}) \right] \right\}.$$

Since the velocity field in the region y > 3 is nearly potential, scattering is determined by disturbance vorticity (2.10).

3. Consider the excitation process of TS-waves on the basis of Eqs. (2.7) and (2.8) in the presence scatter on roughness. The dependence of the growth rate γ of TS-waves (curve 1) and the increment in wave number $\Delta\alpha = \mathrm{Re}\,\alpha - \alpha_c$ (curve 2) on x are schematically shown in Fig. 1. Solving (2.7) with the boundary condition $a(x_0) = 0$ (x_0 is chosen in the damped region sufficiently far from the critical point x_c), we get

$$a = a_{\text{eff}}(x) K(x) \left(K = \exp\left[\int_{x_c}^{x} \gamma dx \right] \right), \qquad a_{\text{eff}} = \int_{x_0}^{x} F^{(1)} (1/K)\, dx. \tag{3.1}$$

Here K(x) is the growth rate of the boundary layer, the factor a_{eff} is the effective (reduced to the critical point) amplitude of the induced wave. The function 1/K within the integrand in (3.1) has a maximum at $x = x_c$ which formally confirms the qualitative description of the formation of the induced field, as shown in Section 1.

Consider scattering on sinusoidal waviness $\eta = \sin k_g x$. In this case Eq. (2.8) is true in a sufficiently small neighborhood of that point where the resonance condition is met. Considering the dominance of the contribution by the region adjacent to the critical point, we limit the analysis to the case of small deviations from resonance at this point ($|\Delta k| \ll \alpha_c$, $\Delta k = k_g - k_\omega - \alpha_c$). Using linear approximation near $x = x_c$, $\gamma = \mu_r(x - x_c)$, $\Delta\alpha = \mu_i(x - x_c)$, where $\mu_r + \mu_i i = i(\partial\alpha/\partial x_c)$ has been introduced. Neglecting the change in σ in the zone of wave excitation and using (3.1) and (2.8) we get

†The stationary phase of the exponential term in (2.8) corresponds to resonance.

$$a_{\text{eff}} = \frac{1}{2i} s\sigma_c d \int\limits_{-\infty}^{\infty} e^{-\frac{\mu\xi^2}{2} + i\Delta k\xi} d\xi = -i\left(\frac{\pi}{2\mu}\right)^{1/2} s\sigma_c d \exp\left[-\frac{(\Delta k)^2}{2\mu}\right], \tag{3.2}$$

where $\sigma_c = \sigma|_{x = x_c}$; $\xi = x - x_c$. Since at $x - x_c > L_*$, $x_c - x_0 > L_*$ the integral weakly depends on x and x_0, the integration limits in (3.1) are extended to infinity. Equation (3.2) shows that the induced field is a maximum when the resonance condition is fulfilled at the critical point ($\Delta k = 0$). The size of the generation zone determined from the condition that a_{eff} is 0.84 of the limiting value (3.2), equals $L_* = (2/|\mu|)^{1/2}$. The presence of μ_i in L_* reflects the fact that the extent of the generation region decreases as a result of waves coming out of resonance. The solution (3.2) is obtained for small ratios $L_*/(x_m - x_c)$. Using the estimates $\mu_r \sim \gamma(x_m)/(x_m - x_c)$, $\mu_i \sim \Delta\alpha(x_m)/(x_m - x_c)$, it is possible to observe that the greater the wave growth and phase incursion in the active zone of the boundary layer, the smaller is this ratio. At frequencies of practical interest this ratio is very small. For example, at $\omega_N|_{x = x_c} = 0.038$, we get $L_*/(x_m - x_c) \simeq 0.35$. It is possible to show that the linear approximation of γ and $\Delta\alpha$ is good right up to $L_* \sim 0.5(x_m - x_c)$. If in the transformation to (3.2) we put $\sigma = \sigma_c + \sigma_x'(x - x_c)$, then the term $\sim\sigma_x'$ for $\Delta k = 0$ does not contribute to a_{eff}. Since the characteristic scale for the change in σ is $x_m - x_c$, the latter denotes that the accuracy of Eq. (3.2) is characterized by the square of the ratio $L_*/(x_m - x_c)$.

Consider briefly the scatter on random one-dimensional roughness described by the spatially homogeneous random function $\eta(x)$ (average ensemble $\langle\eta\rangle = 0$). Using Eqs. (2.8) and (3.1) with the assumption of a narrow generation zone we get

$$\langle |a|^2\rangle^{1/2} = K(x)A_{\text{eff}},$$

where $A_{\text{eff}}^2 = \frac{2\pi}{|\mu|}|\sigma_c|^2 s^2 \int\limits_{-\infty}^{\infty} d\varkappa G(\varkappa + \alpha_c - k_\omega)\exp\left(-\frac{\mu_r\varkappa^2}{|\mu|^2}\right)$, ($G(K)$ is the spectral density $\langle\eta^2\rangle$).

If there is a small change G in the scale le $|\mu|/\sqrt{\mu_r}$ in the neighborhood of resonance, the expression for A_{eff} takes the form

$$A_{\text{eff}}^2 = \frac{2\pi^{3/2}}{\mu_r^{1/2}} s^2 |\sigma_c|^2 G(\alpha_c - k_\omega).$$

The value of A_{eff} does not depend on μ_i, which agrees with the known result from wave theory on the independence of random interaction from phase relations. For the same reason, the size of the excitation zone in the present case is completely determined by the behavior of the increment: $L_* \simeq (1/\mu_r)^{1/2}$. The strength of induced wave is proportional to the spectral strength $\langle\eta^2\rangle$. For a fixed $\langle\eta^2\rangle$ the maximum effect is achieved for the scatter on a wavy surface with a characteristic roughness of the order 1/Re α.

Within the framework of the above-described procedure for the solution of (2.7) and (2.8), it is possible to consider weak flow nonparallelness, by maintaining a constant derivative of δ_* with respect to x and a small transverse component of flow velocity. On the basis of the results of [8] it is possible to show that flow nonparallelness results in a small (of the order of 1/R) complex additive to the increment in (2.7) of quasiparallel theory. Here $F^{(1)}$ practically does not change since the profile φ and the wavy flow field satisfy equations of quasiparallel theory with an accuracy of up to order $\sim 1/R$. The addition to γ depends on the normalization of the profile φ. Further, in computing σ_N we shall use the normalization max $|(1/\alpha_N)\varphi'| = 1$, which determines $|a|$ as the maximum amplitude of streamwise velocity fluctuations on the TS-wave profile. The increment to γ weakly shifts the critical point and in the narrow region of wave excitation leads to small changes in the growth coefficient K of quasiparallel theory. As a result, the changes in amplitude a_{eff} will be small, which should then be considered the effective amplitude reduced to the critical point of the nonparallel flow. Here the critical point is that point at which the maximum amplitude of streamwise velocity begins to grow. It is known that the neutral curve of the boundary layer on a small wavy surface is displaced with respect to its position on a smooth flat plate [10]. This effect can be included in (2.7) if one considers the double scatter of TS-wave on roughness. The change in a_{eff} will also be small in view of the small value of the corresponding increment in γ.

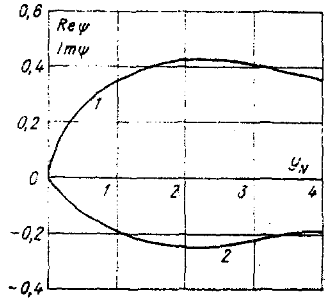

Fig. 2

Let us briefly discuss the question of the limits of applicability of single scatter approximation. Since no slip condition at the surface leads to the formation of a thin viscous layer near it, the limitation on the height of waviness appears to be appreciably stronger than the simple condition of being small with respect to displacement thickness. It is necessary to require that the vertical scale of the velocity field near the surface H is greater than the wave height $h = \max|\eta|$. In the acoustic field and in TS-wave near the surface, viscous layers appear; their scales are identical and equal to $l_w \ll \delta_*$ [6, 11]. Hence it follows that the limitation on the wave height is of the type $h/l_w \ll 1$. In the case of roughnesses with characteristic length $l \leqslant h$, it is not possible to limit to the linear term in the expansion of the flow field about η, since $H \leqslant h$. In the case of smooth nonuniformities ($h \ll l \leqslant 1/\mathrm{Re}\,\alpha$), comparing viscous and inviscid terms of Eq. (2.1), it is possible to introduce a transverse scale for flow field $l_1 = [l\nu/(\partial U/\partial y)_0]^{1/3}$†. It is possible to show that $H \sim l_1$ when $h \leqslant l_1 \leqslant 1$ and $H \sim l$ if $l_1 \geqslant l$.‡

Thus, the limitation on the roughness height associated with the flow takes the form $h/l_1 \ll 1$.

4. In order to compute α_{eff} it is necessary to find μ and derive the functions $\bar{\psi}$, φ, χ. Denote the "current" frequency and wave number of the boundary layer by $\Omega_N = \Omega\delta_*/u_\infty$ and $K_N = k\delta_*$, respectively. Linearizing the dispersion equation of quasiparallel flow theory $\Omega_N = \Omega_N(K_N, R)$ in the neighborhood of the critical point and considering the relation $R - R_c \simeq (3/2)\xi/\delta$, it is possible to obtain

$$\bar{\mu} = \delta_c^2\mu \approx \frac{3}{2}\frac{i}{R_c}\left[\frac{\Omega_N - (\partial\Omega_N/\partial R)R}{(\partial\Omega_N/\partial K_N)} - K_N\right]_c,$$

where the index c means that the expression is taken at the critical point; $\delta_c = \delta_*(x_c)$.

The function $\bar{\Psi}$ is obtained as the sum of "viscous" and "inviscid" solutions (see, similarly [6]). The "viscous" solution has a characteristic length $l_{1N} = (\bar{u}_0{}'\bar{k}_N R)$ and locally close to the surface $y_N = 0$ and the "inviscid" solution is found with ideal fluid approximation ($R = \infty$). Computed results are shown in Fig. 2 for $\bar{\psi}$ at the point $\bar{k}_N = 0.134$, $R = 1620$ (curves 1 and 2 for Re $\bar{\psi}$ and Im $\bar{\psi}$, respectively).

Equations for Φ and χ were solved by the Runge–Kutta method with orthonormalization. Effective amplitude of TS-waves induced at the surface with sinusoidal waviness was sought for exact resonance at the critical point $R_c = 1620$, where $\alpha_N = 0.134$, $\omega_N = 0.038$, $\bar{\mu} = (2.9 - 1.46i) \cdot 10^{-5}$. Computations for acoustic scattering give $\sigma_s = 0.32 + 0.1i$, $\sigma_y = 0.1 - 0.16i$. In order to estimate the effectiveness of vortex scattering the disturbance profile was given in the form

$$\Phi = \frac{b_1}{\sqrt{2}}\exp\left[-\frac{(y_N - b_0)^2}{b_1^2} + \frac{1}{2}\right].$$

Here s in Eq. (2.10) is the amplitude of the maximum streamwise velocity fluctuations and the streamwise velocity profile $|\Phi'|$ qualitatively agrees with the result obtained in [1]. When

†For roughnesses with $l = 1/\mathrm{Re}\,\alpha$ the scale $l_1 \ll \delta_*$ coincides with the scale for the critical layer of TS-wave [6].

‡ When $l_1 \leqslant h$, the quantity H is estimated using the known equation for boundary-layer thickness [12]: $H \sim l/\sqrt{Rl}$ ($Rl = hl(\partial U/\partial y)_0/\nu$ is the Reynolds number based on roughness).

$b_0 = 5$, $b_1 = 1$ for resonant waviness with wave number $\bar{k}_N = 0.1$, the scatter coefficient is $\sigma_N = (4.86 + 0.32i)\cdot 10^{-5}$

The effect of induced wave on the transition to turbulence depends on its amplitude, growth rate, and background noise which causes transition in the absence of the induced wave. According to the data given in [13], the maximum growth for $R_c = 1620$ is e^{10}. We shall consider background noise to be such that at the "natural" transition point the growth rate becomes e^6. In order to shift the transition point it is necessary that the induced field at $K = e^6$ attains the level of strong nonlinearity u_{tr}. According to the measurements in [14], we assume $u_{tr} = 0.02u_\infty$. In order to excite such a wave by scatter on waviness with parameters $d/\delta_c = 0.5 l w/\delta_c = 0.09$, $k_g\delta_c = 0.134$, it is necessary to have noise with amplitude $s = 0.5\cdot 10^{-5}$. The vortex disturbance should have an amplitude $s = 0.05$ with $k_g\delta_c = 0.1$. In particular, if there is an air flow over a flat plate with velocity $u_\infty = 25$ m/sec, it is necessary to have a noise level of 41 dB in order to excite waves with frequency 156 Hz by a scatter on waviness with d = 0.09 mm and period 4.55 cm. In the case of vortex disturbances resonant waviness has a period 6.35 cm.

Thus, scattering of acoustic and vortex disturbances in a boundary layer on a wavy surface can lead to distributed excitation of Tollmien–Schlichting waves. Computations show that even with extremely small waviness, whose height satisfies requirements for single scatter, it is possible to attain induced wave strength sufficient to shift the transition point to turbulence. In the case of scattering on sinusoidal waviness the most effectively excited waves are those for which resonance conditions are fulfilled at the critical point (for a fixed period of waviness, resonance conditions can be satisfied by varying the frequency of external disturbances). Scatter on random waviness is determined by resonant harmonics of its spectrum.

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